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A brief history of the solar oblateness.
A review.

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Abstract

We hereby present a review on solar oblateness measurements. By emphasizing historical data, we illustrate how the discordance between experimental results can lead to substantial improvements in the building of new technical apparatus as well as to the emergence of new ideas to develop new theories. We stress out the need to get accurate data from space to enhance our knowledge of the solar core in order to develop more precise ephemerids and ultimately build possible new gravitational theories.

Key words: Sun: characteristic and properties, 96.60.-j; Solar physics, 96.60.-j; Solar activity, 96.60.Q-; Solar interior, 96.60.Jw; Relativistic astrophysics, 95.30.Sf, 98.80.Jk

1. General purpose

The story of the solar oblateness began in 1865 when Newcomb (55) tried to explain the discrepancy between the prediction of Newtonian gravitational theory and the Mercury perihelion advance anomalies. These anomalies have been observed by Le Verrier in 1844 (49), and measured to be of 43.11"/cy (± 0.45")[1], a value that could not be explained by presence of the known planets. In these pre-relativity days, Newcomb proposed that the value of the solar oblateness Δr, i.e. the difference[2] between the equatorial r_{eq} and polar r_{pol} solar radius, could give an answer to this puzzle. He estimated that if $\Delta r \approx 500$ mas[3], the measured excess motion could be explained within the frame of Newtonian gravitational theory. Such a large oblateness was shortly afterwards rejected on the grounds of observations, as more accurate modern measurements confirm. In 1916, Einstein's theory of gravitation provided for the first time a realistic explanation; it is admitted today that General Relativity (GR) can account for *almost all* the observed perihelion advance *but not all*. The most accurate precession rate of Mercury's perihelion, as currently observed, is 43.13"/cy with a realistic standard error of 0.14"/cy (61), whereas the GR's prediction, according to the most accurate ephemerids, is 42.981"/cy (accuracy on the last decimal), assuming a zero solar oblateness. Once this latter is introduced, the agreement can be carried out to the second digit[4] (60). In modern astrometry, the perihelion advance of planets is still considered as a cornerstone for testing General Relativity (45; 60). Indeed, minor planets, such as Icarus, Ceres or Pallas, for which the orbits can be computed with very high accuracy, present as well an excess motion. Computations for Icarus for instance give 10.058"/cy in GR, perfectly detectable by modern means (60). Still, the contribution of the solar figure to the perihelion shifts of the planets (though less important than first suggested by Newcomb in the case of Mercury) can not be dis-

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[1] "/cy stands for arcsec per century.

[2] Note that the solar oblateness denoted by Δr is a common language misuse, as the oblateness is $\Delta_\odot = \frac{r_{eq}-r_{pol}}{r_{eq}}$ and the flattening $\varepsilon = \frac{r_{eq}-r_{pol}}{r_{R_\odot}}$, where $R_\odot$ indicates the mean solar radius.

[3] mas stands for milliarcsec.

[4] An estimate of Δr of 5×10^{-5}, gives an extra advance of 4.05"/cy, far from the 0.15" needed today.

carded, and must be revisited to the light of new space dedicated missions.

2. Alternative theories to GR

A change in the situation regarding gravitational theories occurred in the early 1960s when Brans and Dicke (12) put forward a scalar-tensor theory of gravity, containing a coupling parameter ω. This factor measures the interaction between a scalar field, added to take into account the Mach's principle and the usual tensor field. In the Brans-Dicke theory, this parameter does not depend on the scalar field, that is to say is constant[5]. The perihelion advance is equal to

$\delta\omega = \left(\frac{4+3\omega}{6+3\omega}\right) \times$ (value predicted by GR),

which required $\omega \geq 6$ to satisfy the observations. For $\omega = 6$, the prediction is 39.6", which is just a bit less than the accepted figure. Dicke though that the difference could be ascribed to a solar oblateness and consequently, set up an instrumentation to test this idea. Results where published in 1967 by Dicke and Goldenberg (22) showing a Δr of 41.9 ± 3.3 mas, allowing a constant coupling of 6.

In spite of the fact that the experiment conducted in Princeton used high technology devices for that time, the publication was received with skepticism and was followed by a lot of criticisms. A number of major papers were published attempting either to find explanation for this large amount of oblateness, or to find arguments to refute it, or to raise questions and doubts about the observations. A brief discussion of the oblateness controversy can be found in (43). One of the main objections bore upon the high value of the oblateness reported, as compared with the theoretical oblateness due to surface rotation alone. Moreover, an oblateness accounting to that extent to Mercury's orbit anomaly would have effects on other planets, but it did not seem to have been fully considered.

To investigate the relativistic issues further, Hill and Stebbins (41) decided to build a dedicated telescope, operating in the hills northwest Tucson (AZ, USA). The experiment called SCLERA[6] lead to a measured Δr of 9.2 ± 6.3 mas (in 1973), roughly a fifth of Dicke-Goldenberg's value. In 1996, Richman (64) propounded a complete review of the two methods, and concluded that the excess found by Dicke and Goldenberg was due to an equatorial excess brightness. Earlier on, in 1972, Chapman and Ingersoll (15; 16) had already suggested that the photospheric faculae near the equatorial solar limb may provide an excess brightness which would explain the Dicke and Goldensberg's oblateness measurement (see section 4.2.1).

The discussion of the results sets *de facto* a fundamental question: what is really a solar diameter? When observing a solar diameter, at any heliographic latitude, what is the observer really measuring? These questions address two key aspects of solar diameter measurement: on one hand, the formal definition of the solar surface and shape from a theoretical and physical grounds, and on the other hand, the definition of reliable and unbiased means to observe it. In the former case, the shape of the Sun can be defined by a surface of constant potential, bearing in mind that the interpretation of the surface shape is complicated by the fact that surfaces of constant density, pressure, and potential do not exactly coincide. This matter will be further addressed in section 5. In that latter case, the solar edge is most commonly defined by the inflection point of the center-to-limb darkening function. Dicke (27) has widely debated on the question both from a theoretical and observational point of view, including the question of an equatorial temperature excess to account for the observed oblateness as an effect of surface field only. Chapman (15) discussed the effect of faculae. Rösch and Yerle (65) have pointed out the effects of the terrestrial atmosphere, scintillation and blurring, which cause a shift of the inflection point[7]. Hill and coworkers (42) have reviewed the different definitions of the solar edge and have proposed to use the finite Fourier transform (FFT) of the observed limb center-to-limb darkening function to achieved reduced sensitivity to atmospheric and instrumental effects. This property means that this function is well suited to detect differences in the polar and equatorial brightness profiles, and hence to discriminate between true oblateness and excess brightness.

While those crucial aspects are still at the heart of solar oblateness measurements debates, the link between oblateness and alternative theories to GR remains relevant. Nowadays, Post-Newtonian theories of gravitation have been developed and spacecraft missions allow to deduced the β and γ parameters with an increased accuracy. Let us recall that β encodes the amount of nonlinearity in the superposition law of gravitation (with

[5]In modern theories, ω is dependent on a parameter ϕ which characterizes the considered tensor-scalar theory. In Brans-Dicke approach, $\phi = 0$. See for more details Will C. (Living reviews in general relativity: http://relativity.livingreviews.org/Articles/lrr-2006-3/).

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[7]This shift is taken into account in the analysis of modern heliometer data, and is quasi identical to the FFT process proposed by Hill (42).

$\beta \equiv 1$ in GR), and contributes to the relativistic precession. The latter parameter γ encodes the amount of curvature of space-time per unit rest mass. In the currently accepted models used for planetary ephemeris, there is a strong correlation between β and the solar gravitational moment of order 2 (called J_2, linked to the oblateness), up to 80% for some data sets (JPL DE405 ephemerides). Hence, using those models to infer the value of both parameters simultaneously leads to a degenerate solution. However, the solar quadrupole moment has other direct measurable influences, as for example, on the value of planetary spins or on the characterization of the ecliptic plane. Finally, through Solar System spin-orbit couplings, the value of J_2 will indirectly influence the orbital parameters of Solar System bodies. For example, the Moon-Earth spin-orbit coupling propagates the influence of the solar quadrupole moment to the Moon. This allowed to set a dynamical upper bound on the solar quadrupole moment, $J_2 \leq 3\,10^{-6}$ (11; 68), using observed lunar librations.

The steadily increasing accuracy in space experiments let us foresee that the days when violations of Einstein's theory may be assessed and understood are drawing near. In recent theoretical developments, a new paradigm, rooted in the role of string theory in early cosmology, has recently emerged (78). The equivalence principle and GR are generically and jointly violated due to a small scalar field with its own dependence on time and space, which is expected to affect all the different physical interactions. While a real computable theory seems at this time difficult to achieve, we have at least a firm prediction that $\gamma < 1$ based on general considerations; proving this constraint wrong would be a major breakthrough.

3. Earlier determination of the solar oblateness

3.1. Poor's results

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One of the first attempt to determine the Figure of the Sun was made by Poor in 1905 (58; 59), who examined hundred photographic plates taken by L. M. Rutherford at the Observatory of Columbia University (USA) between 1860 and 1874. A large number of plates was rejected for various reasons (no marks for the orientation, poorly developed...); only 22 plates were good enough to be used, 4 in 1870, 8 in 1871 and 10 in 1872. By determining the coordinates of the center of the Sun, and after correcting for differential refraction, he was able to measure the most probable value of the Sun's radius. A mean of fourteen polar and equatorial radii was found

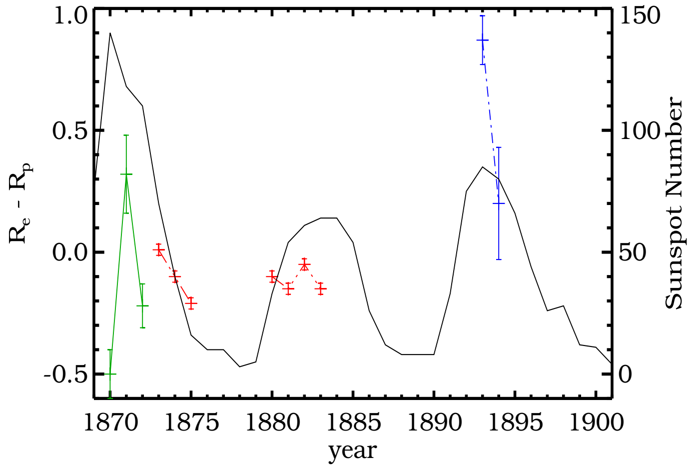

Figure 1: Solar oblateness ($R_{eq} - R_p$, left scale) as deduced from Poor (58; 59) using photographic plates taken at Columbia University (USA) and at the Observatory of Northfield (Minn., USA), and from Ambronn and Schur (3) by means of an heliometer located at Göttingen Observatory (Germany). See text and Table 1 for details. In spite of the fact that measurements are certainly faulty, the general trend indicates a phasing with solar activity (right scale), a rather remarkable result for that time (1905).

for each plate, so that the difference has been formed together with the statistical errors. In a second step, 5 other plates taken at the Observatory of Northfield (Minn., USA) in 1893 (3 plates) and 1894 (2 plates) have been analyzed in the same way. The values of the oblateness obtained are given in Tab.1 and are plotted in Fig. 1 together with the the sunspot activity cycle. Let us quote the conclusions written by Poor: *"the measures seem to indicate a change in the relative sizes of the polar and equatorial radii of the Sun. During this period 1870-1893, there was a real change in the shape of the Sun"* and *"The slopes of the observational curves are nearly parallel to the corresponding portions of the sunspot curve [...]. Ratio between the polar and equatorial radii of the Sun is variable, and the period of variability is the same as the sunspot period"*. Even considering uncertainties, it is notably striking that with the means used at that time the result is rather convincing.

3.2. Auwers' results

While observing the transit of Venus during the years 1874-1882, Auwers (7) made a great number of determinations of the Sun's diameter. A total of 2 692 separate measures of the diameter were made by twenty-three observers. This amount of data was discussed by Auwers who concluded that the diameter of the Sun at

Year	$\Delta r = R_e - R_p$	Remarks	Year	$\Delta r = R_e - R_p$	Remarks
1870	-0.50 ± 0.10	Columbia Univers.	1873	0.01 ± 0.023	Heliometer
1871	0.32 ± 0.16	Columbia Univers.	1874	-0.10 ± 0.023	id
1872	-0.22 ± 0.09	Columbia Univers.	1875	-0.21 ± 0.023	id
1893	0.87 ± 0.10	Northfield plates	1880	-0.10 ± 0.023	id
1894	0.20 ± 0.23	Northfield plates	1881	-0.15 ± 0.023	id
			1882	-0.05 ± 0.023	id
			1883	-0.15 ± 0.023	id

Table 1: Solar oblateness as deduced from Poor's analysis in 1905 (58; 59) (left) and by means of the heliometer by Schur and Ambronn, also published in 1905 (3) (right).

distance unity (AU) is 959.63", and that the polar diameter exceeds the equatorial diameter by (0.038± 0.023)".

This apparent anomaly in the shape of the Sun was explained by Auwers as being due to the tendency on the part of an observer to measure vertical diameters greater than horizontal diameters. Auwers formed the mean of all values and the measurements did not grant any indications of a change of the relative diameters with time. However a change in the differences between equatorial and polar radii can be seen when plotting the measurements years by years on the same graph as those given by Poor. Auwers concluded: *"one settlement is that the change is in the same direction as indicated by the Rutherford plates"*.

3.3. Ambronn and Schur's results

At about the same time, W. Schur and L. Ambronn made a great series of heliometer measurements at the Göttingen Observatory (Germany) in order to get an accurate description of the Sun's shape (2). The program started in 1890 and continued until the end of 1902. A reversing prism was introduced into the heliometer eyepiece so that the measurements of equatorial and polar radii were strictly comparable, at least as far as physiological effects were concerned. On the whole, 163 days of observation were used by Schur and 206 by Ambronn. For the first author, the polar diameter was shorter than the equatorial one in average by 0.007 ± 0.015", and for the second, it was the longer by 0.002 ± 0.009 ". In each case, the difference is smaller than its mean error, so that Ashbrok (6) wrote: *"these numbers indicate that, to within very narrow limits, the Sun is a sphere. Incidentally, it was found that the Sun's diameter remained constant to within about 0.1" in the course of the solar cycle"*. The mean values of the oblateness by year are given in Tab.1 and are as well plotted in Fig. 1 together with the the sunspot activity cycle. Let us remark that Poor (59) rediscussed the Göttingen observations to conclude that the *"heliometer measures thus tend to supplement and confirm the conclusions that the shape of the Sun is variable [...]. The resemblance to the Sun-spots curve is striking"*.

3.4. Critics of historical results

Other determinations of the solar oblateness were made by Chevalier (17) and Hayn (40) using photographic plates and by Meyermann (54) with a heliometer, but all these determinations yield a prolate Sun, which is not correct. The Göttingen program has inspired a great confidence because of the meticulous care with which it was carried out. This is certainly the reason why, from the beginning of this century to the end of the sixties, the adopted geometry of the Sun was merely a sphere. The radius of this sphere is 959.63", as found by Auwers (7) and this value is transcribed in all the editions of the Allen's Astrophysical Quantities. However, the reference to "a circular Sun" disappears after 1963. The difference found by Auwers between the canonical value and the value derived from meridian transits, 1.57", called *"irradiation"* and of which the origin has always remained unclear, also disappears from the Ephemerids around the year 1963. In 1965, the Allen's Physical Quantities gives: *"Semi-diameter of the Sun, circular to ± 0.1"*. The reference to the ellipticity then disappears in the following editions, to be re-introduced in the fourth edition in 2000 as: *"oblateness, semi-diameter equator-pole difference: 0.0086"*.

The difficulties encountered in the analysis of solar disk's astrographic plates are summed up by Wittmann and Débarbat (80); they include the accurate determination of the focal length of the telescope and its possible variation (thermal effects), and the accurate determination of the solar limb, as the profile is a photographic density and not an intensity profile. Most of the measurements gives a difference between equator and pole that is in any case too large to be correct in an absolute way. Even considering the measurements correct up to a magnifying factor, the historical data must be considered with some reserve. A first doubt may rise from the negative differences Δr obtained, which means a prolate spheroid. This shape has certainly no physical interpretation. A second one may come from the use of the heliometer itself: it is essential to keep the image displacement line fixed relative to the observer when rotating the instrument. Some bias could have been introduced, that would yield erroneous measures. Finally, the effect of the fine structure of the Sun's surface on the measurements, such as faculae, is often omitted and could explain some inconsistencies. However, it must be noted that the yearly averages of Δr over the 9 years of Schur's and Ambronn's observations are respectively anticorrelated, with a significant correlation coefficient of -0.65. Thus, it is not surprising that the mean of the two sets converges to 0 (see also Dicke and Goldenberg (22)). Strictly speaking about the analysis, the method used by Auwers is very comparable with the one used by Poor, the difference lies in the degree of confidence the authors give to the standard deviations: according to Auwers, the Sun is spherical within ± 0.1" but according to Poor, the shape of the Sun is cycle dependent. To conclude, let us mention that the oblateness deduced from Ambronn's measurements is $\Delta r/r$ = -7.80 $\times 10^{-6}$ and -6.94 $\times 10^{-6}$ from Poor (as computed from Table 1), which is comparable.

3.5. *Subsequent indirect measurements*

From 1975 to 1986, measurements were made at the Belgrade Observatory with a meridian instrument by S. Sadzakov and M. Dacie (63): they yielded 349 determinations of the equatorial radius (961.2 ± 0.2)" and 433 determinations of the polar radius (961.10 ± 0.54)". Considering the absolute value alone, this leads to an oblateness of 10^{-4}, which is patently unbelievable and gives suspicions about the observations.

The same remark applies for solar astrolabe data, for which it was claimed a solar departure to sphericity. Fig. 2 shows the various astrolabes' measurements plotted as a function of the heliographic latitude, leading to an oblateness of:
1.78×10^{-4} (Chili data from Noël (56)), 8.34×10^{-5} (CERGA data from Laclare (56)) and 5.63×10^{-4} (Sao-Paulo data from Emilio (33)).

These values are still one order of magnitude greater than what is expected, which cast doubts on the validity of such measurements. By contrast, the Rio de Janeiro (62) data yield a polar radius shorter than the equatorial radius by 13 ± 4 mas, but the data analysis process still remains unclear. Note that the large solar radius variations in time (some 0.5" to 0.7" peak to peak amplitude along the cycle) measured by the astrolabes may come from a magnification of the solar signal within the UTLS[8] zone (8; 9). However, Fig. 2 raises an interesting question. If it is obvious that all analyzed solar astrolabe data lead to a value of the solar oblateness that is irreconcilable with estimates deduced by modern techniques, and even with theoretical computations, they nonetheless present a relevant correlation with the temperature excess from pole-to-equator as derived from Kuhn et al. (48). By comparison, the Sun's shape deduced from the Pic du Midi observations (see 4.1) that are in better agreement with modern estimates of the oblateness, shows the same correlation, although with a weaker coefficient. Is it a mere coincidence? How can the agreement between otherwise discrepant measurements be explained?

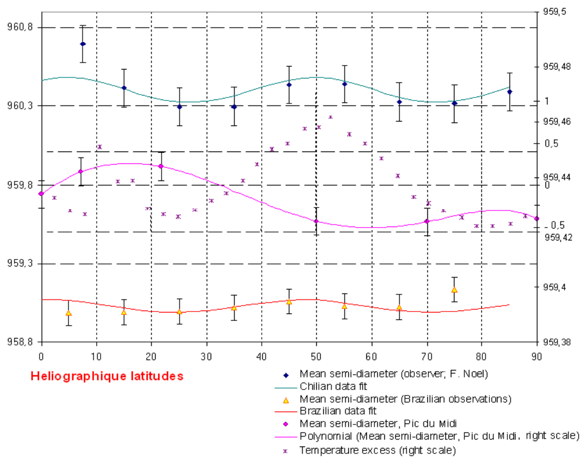

Figure 2: Comparison of the deviation to the mean solar radius obtained by means of the astrolabes, in Chili and Brazil (left scale). The deduced values of oblateness are not consistent with modern measurements. The data obtained at the Pic du Midi observatory (France) plotted versus the heliographic latitude is also shown (right scale); the deduced oblateness is consistent with space data. However, astrolabes data are more correlated with the excess of temperature.

Some other authors tried after the 70's to derive solar oblateness by various means. Let us recall in chronological order, Schatten (72) who, taking again the idea of a rapidly rotating solar core, deduced an *"estimation of the oblateness of the solar photosphere to be near* 3.4 $\times 10^{-5}$ " (flattening). Kislik (47) set an upper limit, showing that the solar oblateness may be neglected in

[8]Upper-Troposphere-Lower-Stratosphere. Could the astrolabes be more useful to probe the properties of this region of the earth's atmosphere than to measure the sun's diameter? It has also been argued that the enhanced solar radius variation observed from astrolabes may come from a stronger magnetic field in the upper layers of the photosphere: see (73) and the discussion in (51).

theories for motion of the inner planets. Burša (13) computed a *"realistic estimation of the Sun's polar flattening"* of 10^{-5}. Finally, Afanaseva et al. (1) determined a solar dynamical oblateness of $(0.66 \pm 0.9)\times 10^{-6}$, through J_2, using radar observations of planets.

The end of the 20th centuries has seen the development of helioseismology, which may help to put new constraints on the solar shape. However the implications of the helioseismic observations for solar radius are complicated by solar structural changes during the activity solar cycle which may also affect the seismic data. Although this point is still not yet clear, the frequency splittings of solar oscillation modes may help to determine the radius of the Sun. Using such a technique, Duvall et al. (31) produced a derived rotation versus depth curve (inversion of the splitting spectrum gave a solar internal rotation approximatively constant with radius down to 0.3 $R_\odot$), yielding $J_2 \times 10^{+6} = 0.17 \pm 0.04$, less than the value obtained for rigid rotation.

4. Modern results, obtained since 1996

4.1. Pic du Midi Heliometer

This instrument was operational at the Pic du Midi Observatory (South France), from 1996 up to 2008. It has been described in (21) and (70)[9]. It operates by fast photoelectric scans of the two solar edges, to "freeze" the atmosphere, and may rotate in all positions around the optical axis. In each measured position, the diameter is determined by building the profile of the Sun in its vicinity. A convolving operation of a theoretical limb profile of the Sun, determined by steps of 1 km (at the wavelength used, i.e. $\approx$ 5300 Å), is made by Gaussian functions of increasing r_0[10] until the fitting adjustment with observed data gives the best correlation coefficient. The interest lies in the fact that r_0 can be determined for each individual scan. The error on each scan is around the mas. Data and results already published in (66), (68) and (71) are listed in Table 2 and are used in Fig. 4.

4.2. Space data

As of today, balloon flights aside, there are very few measurements from space. They can be summarized as follows.

1. Emilio et al. (34) reported a solar shape distortion using the Michelson Doppler Imager (MDI) aboard the

[9]This instrument must be not mixed up with solar astrolabes, working on a completely different principle.

[10]r_0 is the Fried parameter, that accounts for both the instrumental function and the seeing.

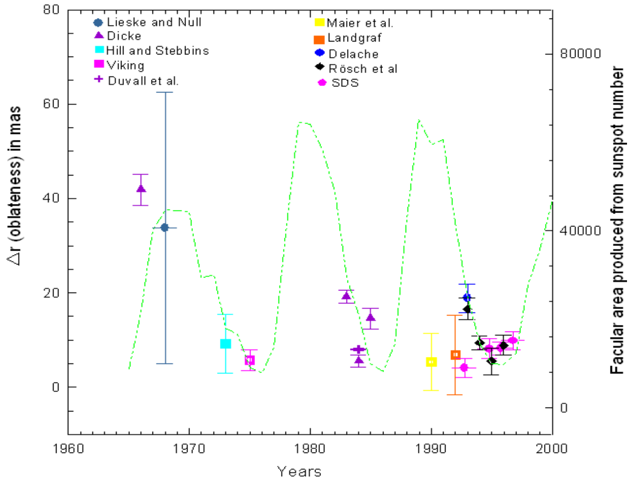

Figure 3: Difference Δr between the equatorial and polar radii according to several authors (see a list of estimates in Godier and Rozelot (37) and here up-graded), plotted versus the faculae area reconstructed from sunspot index. In spite of the (apparent) dispersion, mainly due to the different techniques used, the *observed* oblateness seems to be in phase with the facular area index. Quoted authors are as followed: - Lieske, J.H. & Null, G.W., 1969, ApJ, 74, 297-307. - Hill, H.A. & Stebbins, R.T., 1975-a and 1975-b, Ap. J., 200, 471 and Phys. Rev. Lett., 34, 296. - Viking Probe, reported estimate in 1975 (CNES Report, 1976). - Duvall et al., Nature, 1984, 310, 22. - Delache, P., reported by Landgraf, W., Solar Physics, 1992, 142, 403-406. - Maier, E. et al, Ap J., 1992, 389, 447-452 (can be discarded, see footnote 14).

Solar and Heliospheric Observatory (SOHO) satellite. After correcting measurements for light contamination, they found that the solar shape is nearly purely oblate near solar maximum, but has a significant hexadecapole component near minimum. The deduced oblateness is 8.7 ± 2.8 mas in 1997 and 18.9 ± 1.9 mas in 2001.

2. From observations performed on board the RHESSI satellite[11], Fivian et al. (36) reported an unexpectedly large equator-to-pole radius difference of 10.77 ± 0.44 mas[12], relatively to the one that can be ascribed to surface rotation only (given in the paper as 7.8 mas). Based upon the consideration of the high correlation between the limb position and a sensitive EUV proxy, the 284 Å limb brightness, the authors attribute this excess, dominated by the quadrupole term, to magnetic elements in the enhanced photospheric network. Correcting the computations to be free from magnetic con-

[11]The Reuven Ramaty High-Energy Solar Spectroscopic Imager.

[12]13.72 ± 0.44 mas in (35). Note that the value 10.77 ± 0.44 mas as reported in (36) is of the same order of magnitude that the mean value found at the Pic du Midi for the years 1993-1996: 11.5 ± 3.4 mas, as seen in Table 1 given in (66; 68) (ground-based observations have obviously larger error bars).

tamination, they give a solar oblateness of 8.01 ± 0.14 mas. The RHESSI observations also establishes limits on the quadrupole dependence of brightness (temperature) on position angle, a crucial unknown in precise measurement of the solar oblateness.

4.2.1. Solar oblateness and faculae network

The oblateness observations made at Princeton showed a value about 5 times the value expected under the assumption that the entire Sun is rotating with the angular velocity of the visible layers. To explain this large amount, Chapman and Ingersoll (16), suggested that the observations were the result of an excess brightness at the equatorial solar limb, due to the presence of faculae and sunspots, although Dicke (23) concluded that their effects were insignificant. Such dependence requires that this excess intensity of faculae originates in an optically thin region above the top of the photosphere. To test this idea, Chapman and Ingersoll (16) developed a model which permitted to tabulate a daily oblateness signal based on solar faculae for the year 1966. From the agreement with the Dicke and Goldenberg measurements, they concluded that a sizeable portion of the observed oblateness was the result of photospheric faculae, the mean effect of sunspots being small. They also expected a flattening of about 7.3×10^{-5} for 1967, assuming that the entire signal is caused by faculae.

In a reply Dicke (24) published a statistical analysis of faculae for the year 1966. He concluded that faculae accounted for only 11 percent of the excess oblateness signal (i.e. the signal corrected for surface rotation), or 9 percent of the total oblateness signal. In another paper (25), he discussed the equatorial brightening near the limb associated with an elevated temperature in the upper atmosphere showing that it was not possible to explain the measurements by such a way. He wrote: "*the only presently known acceptable explanation for the observed solar oblateness is the existence of a gravitational quadrupole moment, the latter contributing 80 percent of the oblateness".*

Chapman and Ingersoll (16) contested the Dicke's arguments by developing a statistical analysis based on the fact that in the latter case the measured signal is only subject to error, whereas in the former case, both the measured oblateness signal and the facular signal are subject to error. They concluded that *"faculae may have contributed all of Dicke and Goldenberg's excess oblateness signal".*

5. Theoretical determination of the solar oblateness

The theoretical determination of ϵ is not very simple and requires assumptions that may not be valid in the case of the Sun (such as hydrostatic equilibrium). There are two ways to tackle the question. The first one has been studied by Chandrasekhar (14) for distorted stellar polytropes, modifying the classical spherical Emden's equation for the ellipsoidal case. The second approach is described by Goldreich and Schubert (38) who analyzed the structure equations in terms of perturbation quantities; the method was taken again by Ulrich and Hawkins in 1981 (79). The first theory quoted above is useful to deal with slowly rotating stars, but is not suitable for treating non polytropic stars. The second theory fails when the centripetal acceleration and gravitational acceleration are comparable. Fortunately, in the case of the Sun, a slowly rotating star, the two approaches are not contradictory and produce nearly equivalent results.

The complete set of equations described in (38) can be found in (5) (with a small error corrected). In this last paper, the authors solved the differential set of equations by using vectorial spherical harmonics and they were able to extract a gravitational oblateness giving $J_2 = -0.222 \times 10^{-6}$ (numerical error 0.02×10^{-7}) which implies $\Delta r \approx 8.1$ mas.

The effect of solar core rotation on the surface shape has also been considered by Paternó et al. (57), using accurate measurements of rotational splittings of the lowest degree acoustic modes and SDS data. It was put forward *"that the Sun's core rotates at a rate in between 1.5 and 2 times the surface equatorial angular velocity. Assuming a rigid rotation for the central layers"*, they found *"that an alternate source of the oblateness may be attributed to a magnetic field of the order of 10^5 Gauss in the interior of the Sun".*

Rozelot et al. (69) used the theory of Figures of equilibrium[13] to compute the asphericities of the Sun's shape. Considering the total potential of the Sun as the sum of a gravitational and a rotational term, the latter being complicated by the latitudinal differential rotation

[13] The theory of Figure has been first developed by Wavre in 1932, then by Kopal in 1960 and Modolenvsky in 1988. A complete text is available in Kondratiev's book "Potential theory and figures of equilibrium", 2003, 624. The method consists mainly in developing all the potentials (gravitational, rotational, or even magnetic), under Legendre's polynomials, taking into account the non uniformity of density and rotation inside the bodies. The theory is widely used today by geophysicists. See: (1) Wavre, R.: 1932, "Figures planétaires et géodésie", Gauthiers-Villars ed., Paris. (2) Kopal, Z.: 1960, "Figures of Equilibrium of Celestial Bodies", Univer. of Wisconsin Press, Madison. (3) Modolenvsky, M.S.: 1988, Geodezika i Kartografiya, 5, 11.

both in surface and in depth, they could compute the contributions of each term, noted respectively ϵ_q and ϵ_s to the deviation from sphericity. The measured signal ϵ, i.e. $(\epsilon_q + \epsilon_s)$, is sometimes called the visual oblateness. In fact, in a non relativistic model, ϵ_q is the dynamical Newtonian flattening and the true difficulty is to try to relate dynamical parameters (ϵ_q and ϵ_s) to geometrical observations, that is to say to the flattening f. The authors found a difference between the equatorial and polar radii which *cannot not exceed 11 mas as an upper bound, the most probable value being 8.53 mas*.

• Considering a uniform (rigid) rotation, Chandrasekhar's computations yield a surface flatness f depending on the rotation rate:

$$f = (0.5 + 0.856\rho_m/\rho_c)\alpha \qquad (1)$$

where $\alpha = \Omega^2 R_\odot/g$. The ratio central-to-mean density is ρ_c/ρ_m and g is the surface gravity. For the Sun, the following values may be adopted:
$\Omega = 461.6\ nHz$, $g = 2.74\ 10^4\ cm/sec^2$, $R_\odot = 6.95965\ 10^{10}\ cm$, $\rho_m = 1.409\ g/cm^3$, $\rho_c = 150\ g/cm^3$. Accordingly, it is found $\alpha = 2.17\ 10^{-5}$ and $f = 1.11\ 10^{-5}$.

• For differential rotation, the theory of Figures leads to a flatness of about 8.85×10^{-6}, an estimate slightly lower than the previous (rigid) flatness, which may seem in contradiction with the theoretical work of Maeder (52), predicting an increased oblateness when the star is in differential rotation. This can be explained only if $\partial\Omega/\partial r$ is > 0, a behavior observed at latitudes greater than 50° (10; 50).

6. Temporal dependence of solar oblateness

The Princeton Solar Distorsion Telescope (SDT) was used during the summers of 1966 (at Princeton, USA), then in 1983, 1984 and 1985 in Mount Wilson (CA, USA) after several improvements of the instrument. Results are summarized in (29). The main conclusion is that the measures are consistent, in the sense that, as stated by the authors, *"there is no reason known (to us) why the 1966 result should be defective; so that the quantity* Δr *may vary with the 11.4-yr period of the solar cycle"*.

Hill and Stebbins acknowledged such a variability as early as 1975 (41) and reported a slowly varying oblateness with an average equator-to-pole radius difference of about 7 mas.

In 1996, through observations at the Pic du Midi Observatory, Rösch and Rozelot (66) showed also a temporal dependence of the oblateness, which was found to

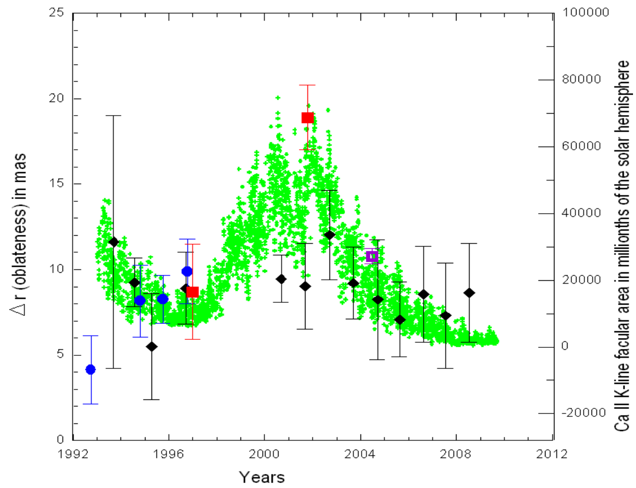

Figure 4: Solar oblateness (difference Δr between the equatorial and polar radius) as deduced from Pic du Midi heliometer observations (lozenges), SDS balloon flights (circles), SOHO-MIDI (squares) and RHESSI space measurements (hollowed out square), plotted together with the faculae area index produced by San Fernando Observatory (US), as a function of time.

be in phase with solar activity. Considering the observations over one solar cycle (1996-2008), the Pic du Midi data still exhibits a relationship with the level of activity on the Sun's surface.

Using the MDI instrument aboard the SOHO satellite and after correcting measurements for light contamination, Emilio et al. (34) reported a solar shape almost purely oblate near solar maximum, and showing a significant hexadecapole shape near minimum, as previously cited (section 4.2).

On the contrary, Egidi et al. (32), and Djafer et al. (30), revisiting balloon flights data recorded in 1992 (September 30), 1994 (September 26), 1995 (October 1) and 1996 (October 10) found an anticorrelation of the solar oblateness with activity. It seems that this tendency is mainly due to a value of $(4.3 \pm 2.0) \times 10^{-6}$ that was taken for the 1992 estimate (see the first SDS point plotted in Fig. 4) which is not in agreement with all theoretical values deduced from solar rotation (and far below, even for the upper bar) [14].

Finally, helioseismology data show a temporal variation in angular momentum at high latitudes (> 45°), through the convection zone, positively correlated with

[14]The data flight from 1990 October 11, as published in Maier (53) has been discarded in (32); according to S. Sofia, this is due to an offset in the data, but it is not impossible that the authors have not measured true perpendicular diameters, thus deducing a smaller value of the oblateness.

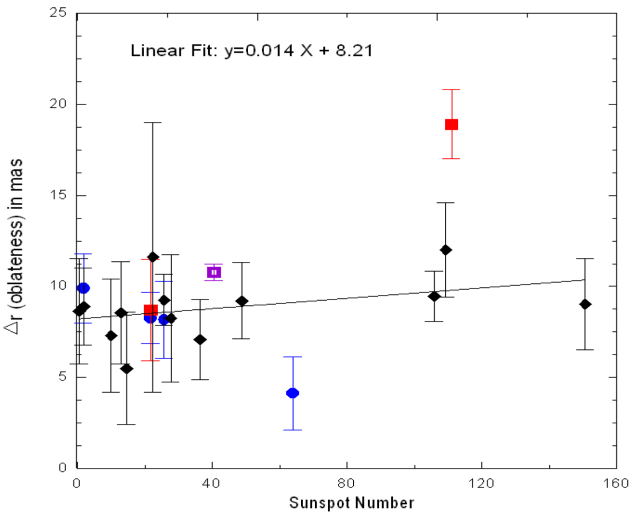

Figure 5: Modern oblateness measurements versus solar activity (International Sunspot Number). For a null activity, the oblateness is 8.21 mas, in good accordance with the theory. Same notations as in Fig. 4: Pic du Midi heliometer observations (lozenges), SDS balloon flights (circles), SOHO-MIDI (squares) and RHESSI space measurements (hollowed out square).

the level of solar activity, whereas at low latitudes it is anticorrelated, except in the 10 % of the radius just below the surface where both are correlated positively (4). This strongly support a higher estimate of the oblateness during periods of higher activity.

In summary, all available data known up to now can be plotted as a function of time. For sake of clarity, we separate data into two groups, one from 1967 up to 1996 (Fig. 3) and the other from that date up to 2009 (Fig. 4). In both figures, the left scale gives the difference Δr between the equatorial and polar radii and the right scale refers to the facular area.

In Fig. 3, the oblateness is plotted according to several authors for which a list of estimates can be found in Godier and Rozelot (37). The dotted line is the facular area reconstructed from the International Sunspot Index. In Fig. 4, the oblateness is plotted versus the Ca-II K line facular area (in millionth of the solar hemisphere) given by the San Fernando Observatory.

In spite of the (apparent) dispersion, mainly due to the different techniques used, the *observed* oblateness seems to follow the behavior of the facular activity index, which is strongly correlated with activity. Finally, all the modern available data can be plotted versus solar activity: Fig. 5, reveals a positive trend for which the extrapolation to a null activity gives $\Delta r = 8.21$ mas ($\pm \approx 1$ mas), in good accordance with the theory. New space data are required to sharpen and confirm this last value.

Does that mean that the measured equatorial diameter is greater due to the presence of an enhanced magnetic network alone? In short, what is the percentage due to the true gravitational oblateness?

To interpret this variability of oblateness in time, Rozelot et al. (71) have conjectured a mechanism that would change the relative importance of quadrupolar and hexadecapolar terms during the course of the activity cycle. In times of high activity, only the first moment has a significant contribution, but in times of low activity, the second one is predominant; this results in a decrease of the total value of the oblateness. The combination of the two terms leads to a solar oblateness varying along solar activity. In our opinion, the presence of a solid rigid rotating core would contribute to the measured excess oblateness. Efforts must be made in that direction by new space measurements.

7. Revealing Sun's interior

In absence of strong magnetic fields, or other internal stresses, the Sun would round up to give a small oblate spheroid as the shape of minimum energy. All departures have to be rationalized. A "static" oblateness has been argued by Dicke: the solar distortion would come from a core rotating rigidly with a period of about 12 days (26; 28). Several criticisms have been made: such a core is unlikely because of the loss of angular momentum by the so-called Ekman pumping (44) or through thermally driven turbulence, or because differentially rotating solar models are unstable (38).

Current investigations of the properties of the solar interior are providing additional indirect support, both experimental and theoretical, for the hypothesis that the Sun contains a decoupled rapidly rotating core (76), likely 1.13 times the mean velocity rate at the surface (i.e. around 450-500 nHz). On another hand, recent time-frequency analysis of solar neutrino data (74) suggests that the core is in substantially rigid rotation with a sideral rotation frequency close to 13.87 yr^{-1} (440 nHz). Using the rotational splitting of the non radial *p*-modes of the global oscillation of the Sun, Isaak (46) concludes on the possible existence of an oblique magnetic rotator inside the Sun. Modern helioseismic measurements should eventually place useful limits on the strength and uniformity of the internal magnetic fields which may be responsible for the surface changes on different oscillation modes.

8. Conclusion

Discussion of historical data, up to around 1996, is certainly more an interesting tour through different techniques than an accurate determination of changes of the solar oblateness, as a physical measurement is really only as good as its uncertainties. Let us point out that the require precision is at the cutting edge of modern available techniques and further progresses will come from dedicated space experiments. The main conclusions from this brief review can be summarized as followed.

- Firstly, Fig. 3 and 4 may suggest that the measured solar oblateness does not indicate a true gravitational oblateness, but may be simply the result of an excess brightness at the equatorial solar limb, due to the presence of photospheric faculae or enhanced magnetic elements in the photospheric network.

- Secondly, we may also wonder if the excess of oblateness shown in measured data, is not merely the increase of oblateness due to the differential rotation, as first suggested by the theoretical simulation made by Tassoul (75). In other words, the excess could be explained by the combined contribution of the oblateness (quadrupolar term) and the hexadecapolar term. This last one would be responsible for a complex outer shape (originating in the leptocline), showing a bulge from the equator up to the top of the royal zones (active latitudes $\pm$ 45°) and a depletion at higher latitudes (the whole shape remaining oblate), and due to an inversion in the radial gradient of the rotation velocity rate at about (45-50)° in latitude ($d\Omega/dr$ is positive beyond these latitudes, negative below).

- Third, we think that what we learned from the solar oblateness can be easily transfered to other stars. We are currently trying to pave the way in this direction (19; 20).

- Lastly, present-day ephemerids have reached the accuracy which allows to test a number of proposed possible modifications to the presently accepted laws of gravitation. In the future, with further improved accuracies, the ephemerids will be able to test even more subtle modifications. It is thus expected to assess the gravitational oblateness of the Sun with an accuracy better than 10^{-8}. The measurement of the relativistic parameters β and γ (which are equal to 1 in Einstein's theory) is inextricably connected with the solar quadrupole moment J_2 which contributes, just as the relativistic corrections do, to the advance of Mercury's perihelion. The expected accuracy for J_2 (10^{-8}) will provide information about the rotation of the solar core and will be relevant for a better understanding of the deep interior of the Sun.

Accurate measurements from space observations are needed. They can be achieved by next generation of satellites, such as PICARD, for which one of the author was one of the first initiators of this program (18), DynaMICCS (77) of which the final aim is to reveal a complete 3D vision of the Sun, SDO (Solar Dynamics Observatory) with a dedicated instrument HMI (successfully launched February, 12th, 2010), or even balloon flights (73) (new flights are scheduled by 2010-2011). And if accepted, new results might come from a Brazilian mission called SPHERIS, specifically designed for astrometric measurements of the solar radius, at a 2020 horizon. To be exhaustive, let us mention the ASTROMETRIA project, from the Pulkovo (Russia) Observatory (39). In the long term, GAIA, a space mission expected to flight by the end of 2012, will allow to estimate the perihelion precession of Mercury, Icarus, Talos and Phaeton. It will be possible to separate the relativistic and the solar contributions in the perihelion advance of such planets, so that gravitational moments could directly be determined from dynamics, without the need of a solar model.

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Year	Weighted Value	Weighted error
1993	11.61	7.41
1994	9.25	1.41
1995	5.5	3.1
1996	8.9	2.1
2000	9.46	1.38
2001	9.03	2.5
2002 (2002.675)	12.02	2.6
2003 (2003.674)	9.21	2.1
2004 (2004.684)	8.24	3.5
2005 (2005.648)	7.08	2.2
2006 (2006.658)	8.56	2.8
2007 (2007.543)	7.3	3.1
2008 (2008.621)	8.65	2.9
Non weighted values are as followed:		
1993.563	23.4	7.4
1993.698	9.8	2.9
1994.437	5.8	3.1
1994.439	10.8	3.6
1994.554	12.6	3.2
1994.560	5.6	2.9
1994.565	12.6	3.1
1995.307	5.5	3.1
1996.693	8.9	2.1
2000.684	9.03	2.5
2001.673	7.43	4.0
2001.676	8.44	2.9
2001.678	7.65	2.9
2001.681	11.49	2.1

Table 2: Solar oblateness as deduced from observations made at the Pic du Midi observatory (F) by means of the scanning heliometer (21). Non weighted values are also available in (66; 68). Since 2002, observations have been made only during a single campaign.

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